## Sampling Methods for Statistical Moments Assessment of

## **Performance Functions**

## Jun Xu

## Professor Hunan University

This series of reports provides an in-depth exploration of density estimation methods in structural reliability analysis, focusing on two key challenges in the field: (1) how to efficiently compute statistical features, and (2) how to accurately reconstruct probability density functions. In addressing the challenge of efficient statistical feature computation, the presentation comprehensively examines methods for efficiently and accurately extracting the statistical characteristics of performance functions from a limited set of data samples. These features include central moments, fractional moments, moment-generating functions, and other essential statistical quantities that are crucial for understanding and predicting the probabilistic behavior of structures.

The presentation begins with a detailed introduction to several multidimensional Gaussian numerical integration methods, including the tensor product method, sparse grid method, cubature formula method, and rotated quasi-symmetric point method. Each method is characterized by unique computational principles and application advantages, making them suitable for a wide range of complex numerical integration scenarios. Following this, the presentation systematically introduces several widely used dimension-reduction techniques. These include the Univariate Dimension-Reduction Method (UDRM), the Bivariate Dimension-Reduction Method (BDRM), and their improved variants. Additionally, the presentation delves into a generalized dimension-reduction technique based on contribution analysis, designed to efficiently compute the statistical moments of performance functions. Finally, the report introduces advanced sampling methods for high-dimensional numerical integration, with a particular emphasis on deterministic sampling strategies.