Non-parametric Distribution Reconstruction Methods for Uncertainty Propagation and Reliability Assessment

Jun Xu
Professor
Hunan University

Nonparametric density estimation methods are a class of statistical techniques used to estimate unknown probability distributions. Unlike traditional parametric methods, which require specific assumptions about the form of the data distribution (such as normal or exponential distributions), nonparametric methods derive the unknown distribution directly from the sample data. This makes them particularly well-suited for situations where the data distribution is complex or unknown. Common nonparametric density estimation methods include Kernel Density Estimation (KDE) and the Mellin Transform. However, these methods often struggle to maintain accuracy, particularly when reconstructing the tails of the distribution.

In this presentation, we will first provide a brief overview of two widely used nonparametric density estimation methods: Kernel Density Estimation and the Mellin Transform. To enhance accuracy, we will introduce an Adaptive Gaussian Mixture Model (GMM) method. This approach leverages the inverse Fourier relationship between the Characteristic Function (CF) and the Probability Density Function (PDF), combined with a convolution search technique for parameter estimation. Initially, a more accurate expression for the CF is derived, with undetermined parameters specified based on a numerically estimated CF curve. Subsequently, a convolution search domain is developed to determine key parameters, including weight coefficients, the mean domain, and the standard deviation domain. Unlike conventional parameter estimation methods, the convolution search technique effectively mitigates issues such as overfitting and sensitivity to initial parameter choices. Using these optimized parameters, the PDF is reconstructed, evolving into an Adaptive Gaussian Mixture Model. Additionally, this presentation will discuss several other recently developed nonparametric distribution reconstruction methods, highlighting their advantages and potential applications.